

WHERE SHOULD THEY HOLD THE FUNDRAISING PARTY?

Jennifer, Steve, and their committee are planning a fundraising event to raise money for charity. They want to rent a hall and provide food and entertainment. Everyone in attendance will pay an admission fee to cover costs and the profit will be contributed to a local charity. To minimize the cost of the hall, Jennifer and Steve call three establishments to obtain quotes. In this unit, you will analyze the three quotes that they receive and answer the question, *Where should they hold the fundraising party?*



DISCUSS

- 1 Explain what is meant by “a minimum total charge of \$400.”

WHERE SHOULD THEY HOLD THE FUNDRAISING PARTY?

Computing Costs at the Galaxy Inn

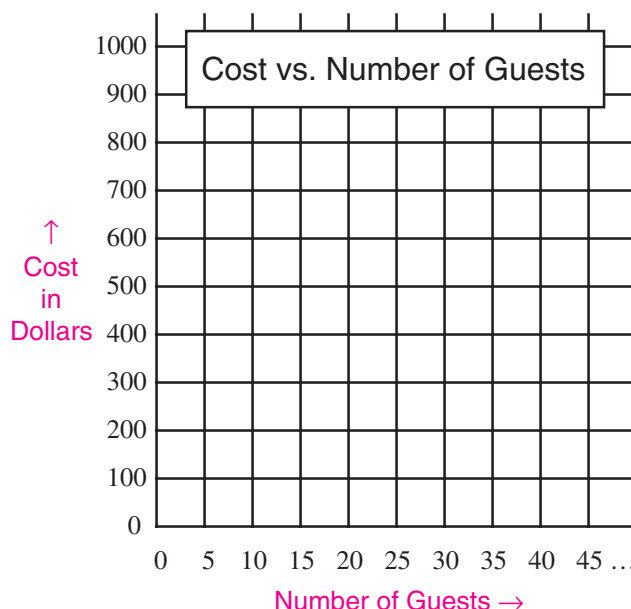
All the exercises below pertain to the quote from the Galaxy Inn.

- ② How much would the Galaxy Inn charge for:
- a) 10 guests? b) 20 guests? c) 30 guests?
- ③ a) How many guests would incur a total cost of \$901?
 b) What is the largest number of guests that can be accommodated for \$1000?
 c) How many guests would it take to achieve an average cost of \$17 per guest?
- ④ a) Make a table showing the cost for each number of guests between 25 and 35.
 b) Use your table to check your answer to Exercise ② c).

Number of Guests	Cost
25	
26	
•	
•	
•	
34	
35	

- ⑤ a) Describe how you could compute the cost for any given number of guests (greater than 23).
 b) Let n represent the number of guests attending the party. Write an expression for the cost for n guests (where $n > 23$).
 c) Use your expression in ⑤ b) to calculate the cost for:
 (i) 45 guests. (ii) 55 guests. (iii) 65 guests.
 d) Use your expression in ⑤ b) to calculate how many guests would cost: (i) \$561. (ii) \$884. (iii) \$1156.

- ⑥ a) Using a full sheet of squared paper, make a graph showing the cost of n guests for these values of n : 20, 25, 30, 40, 50, 60, 70. Save your graph. You will need it in the next activities.
 b) Join the dots in your graph. Write a sentence to describe any pattern in your graph.
 c) Use your graph in ⑥ b) to estimate the cost for:
 (i) 45 guests. (ii) 55 guests. (iii) 65 guests.
 Compare these answers with ⑤ c).
 d) Use your graph in ⑥ b) to estimate how many guests would cost:
 (i) \$561. (ii) \$884. (iii) \$1156.
 Compare these answers with ⑤ d).



X	Y1
42	714
43	731
44	748
45	765
46	782
47	799
48	816
X=42	

- ⑦ Using your graphing calculator, display a table of values for $Y = 17X$ and show how we can scroll through a table of values to check answers. Then graph this function on the screen and trace along it to show how it matches the values in the table.

UNIT 2

ANSWER KEY FOR ACTIVITY 1

- ① A “minimum total charge of \$400” means that no matter how many guests there are, the total charge will not be less than \$400.
- ② a) Cost for 10 guests would be \$400.
b) Cost for 20 guests would be \$400.
c) Cost for 30 guests would be $17 \times 30 = \$510$.
- ⑤ a) Multiply the number of guests by 17 to obtain the cost in dollars.
b) Cost in dollars = $17n$ for $n > 23$
c) (i) $17 \times 45 = \$765$
(ii) $17 \times 55 = \$935$
(iii) $17 \times 65 = \$1105$
d) (i) 33 guests (ii) 52 guests (iii) 68 guests

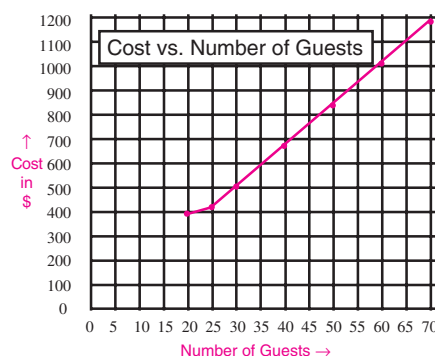
- ③ a) $901 \div 17 = 53$, so 53 guests would cost \$901.
b) $1000 \div 17 = 58.82\dots$, so \$1000 is enough to pay for 58 guests but not for 59 or more.
c) The cost at the Galaxy Inn is \$17 per person when the total cost is at least \$400. This occurs when the number of guests is greater than or equal to $400 \div 17$; i.e., when the number of guests is 24 or more.

- ④ a) The table is shown here.

Number of Guests	Cost
25	425
26	442
27	459
28	476
29	493
30	510
31	527
32	544
33	561
34	578
35	595

- b) We observe from the table that the cost corresponding to 30 guests is \$510. This verifies our answer to ② c).

- ⑥ a) The graph should look like this.



Note: The cost for 23 guests is \$400, for 24 guests it is \$408, and it increases from there at \$17 per guest. Whether the students graph the cost for $n = 20, 25, 30 \dots$ or for $n = 20, 21, 22, 23, 24, 25, \dots$ the graph will be composed of two line segments of different slope.

- b) The dots for $n \geq 24$ lie along a straight line.
c) The students' estimates should be close to \$765, \$935, and \$1105 respectively, as obtained in Exercise ⑤ c).
d) The students' estimates should be close to 33 guests, 52 guests, and 68 guests respectively, as in Exercise ⑤ d).

The NCTM in its discussion draft of *Principles and Standards of School Mathematics* asserts (p. 39):

Rubrics that help teachers analyze learning tasks and describe levels of proficiency can be used to help students understand what performance is expected and how that performance will be evaluated. They provide guidance for students' self-assessment and increase the student's autonomy as they advance through the pre-K–12 years.

The scoring guide presented below has been developed using student responses on extensive field tests conducted across Ontario in 1998–99. It is presented here as a starting point for you in the development of your own scoring guide that will evolve as you work with your students.

The four levels span the spectrum of achievement from very low (Level 1) to exceptionally strong (Level 4). Level 3 represents “good performance” and each student who reaches this level is deemed competent to proceed to the next phase of instruction. Most students are expected to attain Level 3, and for this reason, the scoring guides in this module shade the criteria in the Level 3 column and often express performance at other levels in terms of Level 3 achievement.

Scoring Guide for Activity 1				
	Level 1	Level 2	Level 3	Level 4
REPRESENTATION				
Representation of a Linear Function by: <ul style="list-style-type: none"> • a table • a linear equation • a graph (exercises 4 – 6) A3, A6, A7, A8	•Two or more of the three representations (table of values, equation, or graph) of the Galaxy Inn cost function are missing or incorrect.	•One of the three representations (table of values, equation, or graph) for the Galaxy Inn cost function is missing or incorrect.	•The correct table of values, equation, and graph are given for the Galaxy Inn cost function, with a few minor errors.	•The correct table of values, equation, and graph are given for the Galaxy Inn cost function, with almost no errors.

ACHIEVEMENT LEVELS CORRESPONDING TO THE NCTM STANDARD

- | | |
|----------------|--|
| Level 1 | Identifies achievement that falls much below the standard. |
| Level 2 | Identifies achievement that approaches the standard. |
| Level 3 | Identifies a high level of attainment of the standard. Students achieving at Level 3 can be confident that they are prepared to proceed to the next learning activity. |
| Level 4 | Identifies achievement that surpasses normal expectations pertaining to the standard. |

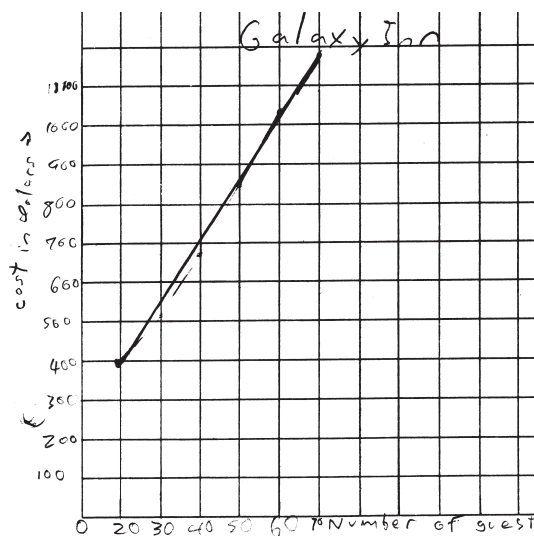
WHAT YOU MIGHT SEE

REPRESENTATION OF A LINEAR FUNCTION BY A TABLE, AN EQUATION & A GRAPH

Level 1

4 a) Galaxy Inn

Number of Guests	Cost
25	425
26	442
27	459
28	476
29	493
30	510
31	527
32	544
33	561
34	578
35	595

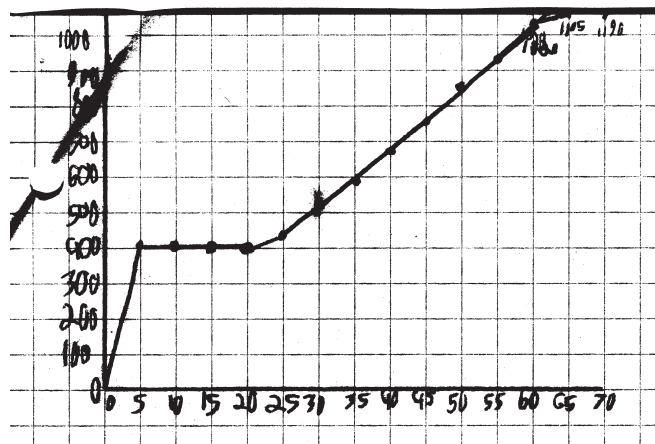


The student has completed the table of values correctly. The graph shows no understanding of the condition that there is a minimum charge of \$400, and shows a charge of \$400 beginning at 20 guests. Also the student was unable to write the cost function $17n$ in Exercise 5 b). Only one of the three representations of the Galaxy Inn cost function was completed correctly.

Level 2

Galaxy Inn

Number of Guests	Cost
25	\$425
26	\$442
27	\$459
28	\$476
29	\$493
30	\$510
31	\$527
32	\$544
33	\$561
34	\$578
35	\$595



The student has completed the table of values correctly. The graph shows an understanding of the condition that there is a minimum charge of \$400, however, the graph is incorrect in the range between 0 and 5 guests, and shows a charge of \$400 beginning at 5 guests. Furthermore the axes of the graph are not labeled. In addition, the student was unable to write the cost function $17n$ in Exercise 5 b). Two of the three representations of the Galaxy Inn cost function were completed, albeit with some minor errors.

WHAT YOU MIGHT SEE

REPRESENTATION OF A LINEAR FUNCTION BY A TABLE, AN EQUATION & A GRAPH

Level 3

Activity 1

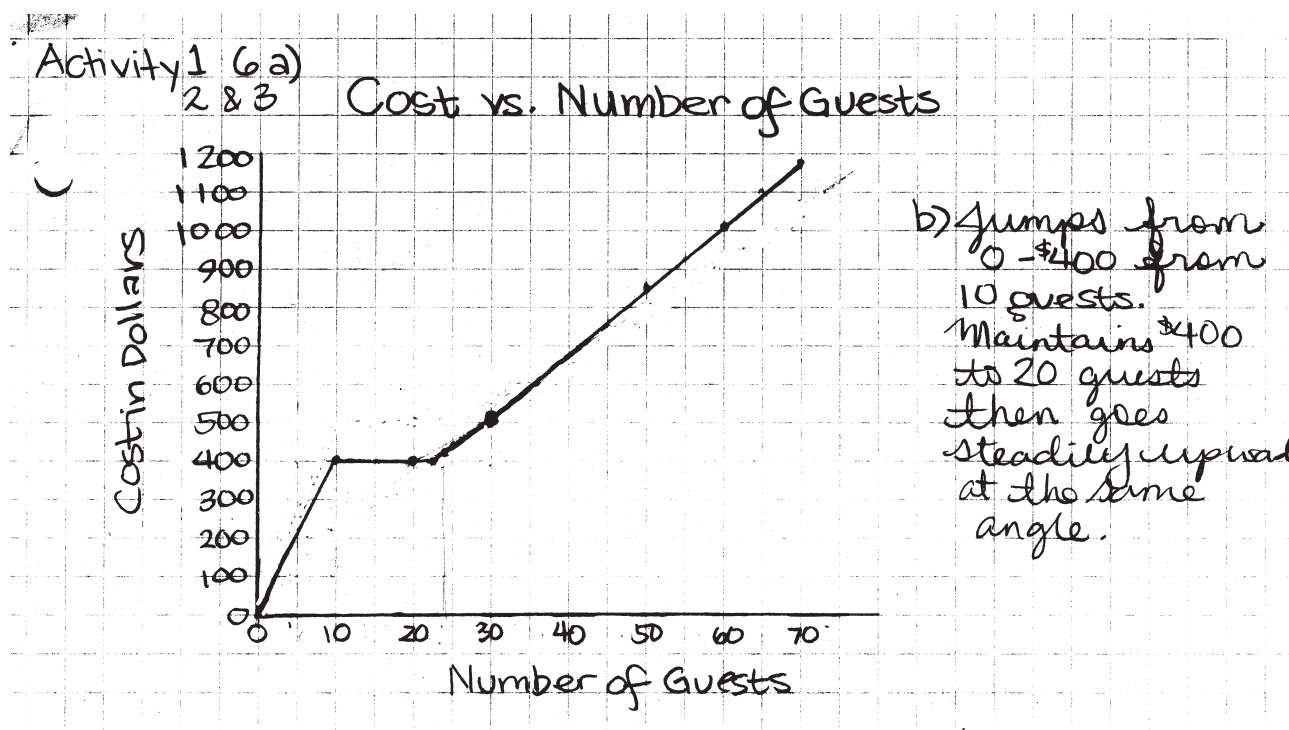
4a)

# of Guests	Cost (\$)
25	\$425
26	\$442
27	\$459
28	\$476
29	\$493
30	\$510
31	\$527
32	\$544
33	\$561
34	\$578
35	\$595

Galaxy Inn

5. a) Describe how you could compute the cost for any given number of guests (greater than 23). $n = \text{number of guests}$ $17n$
- b) Let n represent the number of guests attending the party. Write an expression for the cost for n guests (where $n > 23$). $17n$

↑
The student wrote the answers to this exercise on the sheet.



The student has completed the table of values correctly. The graph shows an understanding of the condition that there is a minimum charge of \$400, however, the graph is incorrect in the range between 0 and 10 guests, and shows a charge of \$400 beginning at 10 guests. It also begins to increase beyond \$400 at 24 guests. The student was able to write the cost function $17n$ in Exercise 5 b) as shown above. All three of the representations of the Galaxy Inn cost function were completed, albeit with a few minor errors.

ACTIVITY 2 – TEACHER EDITION

COMPARING THE GALAXY INN WITH THE NOBLE PINES COUNTRY CLUB

Expectations Addressed

- A3 represent a variety of relations and functions with tables, graphs, verbal rules, and, when possible, symbolic rules;
- A4 develop a sound conceptual understanding of equation and of variable;
- A5 explore relationships between symbolic expressions and graphs, paying particular attention to the horizontal and vertical intercepts, points of intersection, and slope (for linear relations);
- A6 become fluent in generating equivalent expressions for simple algebraic expressions and in solving linear equations and inequalities;
- A7 use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships;
- A8 model and solve contextualized problems using various representations, such as graphs and tables, to understand the purpose and utility of each representation.

Context

This activity extends the investigation of linear functions in Activity 1 from the exploration of the cost function $17n$, where n denotes the number of guests, to the exploration of the cost function $14n + 90$. After students explain in words how to compute the costs for various numbers of guests at the Noble Pines Country Club, they are asked in Exercise 5b) to translate this verbal description into an algebraic expression. In Exercise 6, students graph the linear function $14n + 90$ on the grid used in Activity 1 for the function $17n$. This enables them to compare the graphs of the cost functions of the Galaxy Inn and the Noble Pines Country Club. In Exercise 6d), they are asked to indicate the value of n at which the graphs cross. They are expected to observe that the graphs cross at $n = 30$, indicating that the costs for both sites are equal for this number of guests. This is tantamount to asking for the graphical solution of the linear equation $17n = 14n + 90$. A formal algebraic solution of this equation is not expected at this point.

The purpose of Activity 2 is to help students create and interpret the tabular and the graphical representations of linear functions. They will observe that adding a flat fee makes that option a more expensive proposition for small values of n , but for a sufficiently large value of n , the per person cost (coefficient of n) is more important. Another objective of this lesson is to help students use inequalities to determine the range in the values of n for which each option is preferred. The cartoon on page 70 is intended to clarify for students the meaning of the “best option.” That is, the least expensive option is that which renders the lowest total cost, and this is the same as the lowest cost per guest.

ACTIVITY 2 – STUDENT PAGE

COMPARING THE GALAXY INN WITH THE NOBLE PINES COUNTRY CLUB



ACTIVITY 2 – TEACHER EDITION

The Lesson Launch 15 minutes

Before launching this lesson, ask whether anyone was successful in solving the problem handed out at the end of the previous class (problem 1, p. 18). Display a copy of the problem on the overhead projector and discuss what it means when a two-pan scale is in balance. Ask students,

- Give an algebraic expression for the total mass of the coins on the left pan; on the right pan.
- The two pans are in balance; what does that tell us?
- How would you state this algebraically?
- Finding the value of n that makes both sides equal is called “solving for n ”; how could we do this?

Allow the students to struggle with the last question before providing a hint such as, *How much greater in mass are the n coins on the left pan than the n coins on the right pan?* and *If the n coins on the left pan are $3n$ grams more than the n coins on the right pan, then why are the pans in balance?* Explain to students that they will be solving the problem in a different way as they work through this activity.

To launch the lesson, distribute student pages 70 and 71 and have students read to themselves the cartoon on page 70. Then ask students whether the boy is using an appropriate method for determining the “best deal.” Ask why or why not.

Paired Activity 30 minutes

Group the students in pairs. Have each pair complete Exercises 1 through 5 with each student recording the answers in his notebook. Ask students, when finished, to complete Exercise 6 on the grid they used in Activity 1. When almost all students are finished, ask about the value of n at which the graphs cross. Ask what this means about the costs of the two different sites. Ensure that students understand not only that this is the value of n for which the two sites have the same cost, but also that as the number of guests is increased beyond this, the Noble Pines Country Club offers the lower cost.

If students have access to a TI-73, TI-83, or CASIO *fx-7400G* graphing calculator, ask them to define $Y_1 = 17X$ and $Y_2 = 14X + 90$ and display a table of values showing Y_1 and Y_2 as functions of X . Have them scroll through the table to check their answers to Exercises 5 c) and 6 d). Ask them to graph these functions and trace along the graphs to check their answers to Exercise 6 d).

Closure

If you have a graphing calculator viewscreen, display a table of values for Y_1 and Y_2 and show how we can scroll through a table of values to find n for which $Y_1 = Y_2$. Then graph these functions on the screen and trace to the point of intersection to verify that $Y_1 = Y_2$ for $n = 30$. Discuss why solving for n to make $Y_1 = Y_2$ is the same problem as the two-pan balance problem presented above. Then distribute problem 2 on page 18 and assign it as a homework challenge. As with problem 1, encourage them to involve their parents in the solution.

ACTIVITY 2 – STUDENT PAGE

COMPARING THE GALAXY INN WITH THE NOBLE PINES COUNTRY CLUB

All the exercises in this activity pertain to the quote from the Noble Pines Country Club.

- 1 Explain what is meant by “a flat fee of \$90”. How is this different from a minimum charge?
- 2 How much would the Noble Pines Country Club charge for:
 - a) 10 guests?
 - b) 20 guests?
 - c) 30 guests?
- 3 a) How many guests would incur a total cost of \$566?
 b) What is the largest number of guests that can be accommodated for \$1000?
 c) How many guests would it take to achieve an average cost of \$17 per guest?
- 4 a) Make a table showing the cost for each number of guests between 25 and 35.
 b) Use your table to check your answer to Exercise 3 a).
 c) Compare your table with the table you constructed in Exercise 4 of Activity 1. How many guests are needed to make the Noble Pines cost per guest less than the Galaxy Inn?
- 5 a) Describe how you could compute the cost for any given number of guests.
 b) Let n represent the number of guests attending the party. Write an expression for the cost for n guests.
 c) Use your expression in 5 b) to calculate the cost for:
 - (i) 40 guests.
 - (ii) 50 guests.
 - (iii) 55 guests.
 d) Use your expression in 5 b) to calculate how many guests would cost:
 - (i) \$692.
 - (ii) \$888.
 - (iii) \$1042.
- 6 The graph you constructed in Activity 1 should look like this.

 - a) Using the same grid, plot points in a different color to show the cost at Noble Pines for n guests for these values of n : 20, 25, 30, 40, 50, 60, 70. Join the new dots in your graph.
 - b) Use your graph to check your answers to check your answers to 5 c) and 5 d).
 - c) Label your graphs “Galaxy Inn” and “Noble Pines”. Describe how the graphs are alike and how they are different.
 - d) At what value of n do the graphs cross? What do you think this means?
- 7 Which location, the Galaxy Inn or the Noble Pines Country Club, do you think will offer the lowest total cost for the fundraising party? Explain.

X	Y ₁	Y ₂
20	340	370
21	357	391
22	374	412
23	391	433
24	408	454
25	425	475
26	442	496
27	459	517
28	476	538
29	493	559
30	510	580
31	527	601
32	544	622
33	561	643
34	578	664
35	595	685
36	612	706

X=30

COMPARING THE GALAXY INN WITH THE NOBLE PINES COUNTRY CLUB



DISCUSS

All the exercises in this activity pertain to the quote from the Noble Pines Country Club.

- ❶ Explain what is meant by “a flat fee of \$90.” How is this different from a minimum charge?
- ❷ How much would the Noble Pines Country Club charge for:
 - a) 10 guests?
 - b) 20 guests?
 - c) 30 guests?
- ❸
 - a) How many guests would incur a total cost of \$566?
 - b) What is the largest number of guests that can be accommodated for \$1000?
 - c) How many guests would it take to achieve an average cost of \$17 per guest?

ACTIVITY 2 – STUDENT PAGE

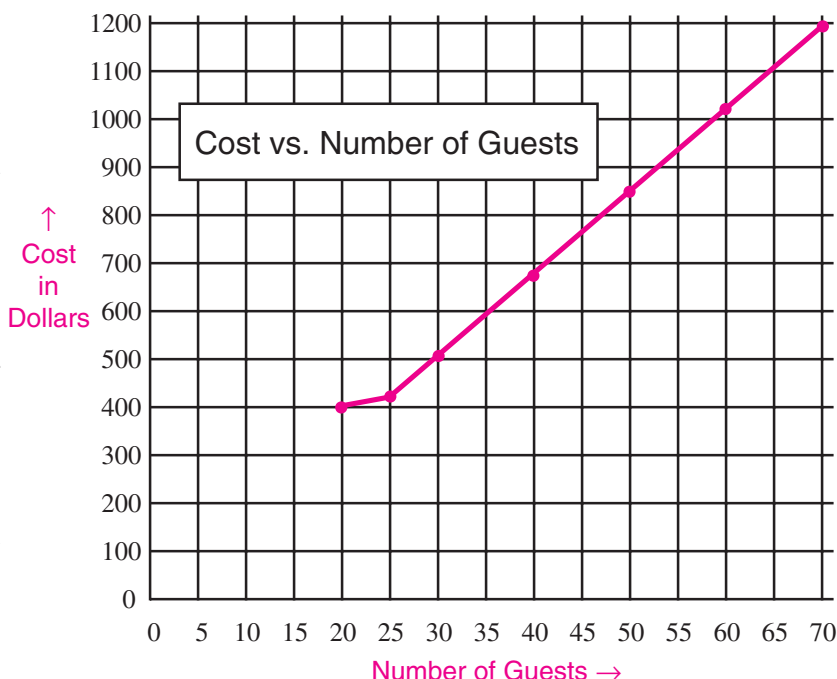
All the exercises below pertain to the quote from the Noble Pines Country Club.

- 4 a) Make a table showing the cost for each number of guests between 25 and 35.
 b) Use your table to check your answer to Exercise 3 a).
 c) Compare your table with the table you constructed in Exercise 4 of Activity 1. How many guests are needed to make the Noble Pines cost per guest less than the Galaxy Inn?
- 5 a) Describe how you could compute the cost for any given number of guests.
 b) Let n represent the number of guests attending the party. Write an expression for the cost for n guests.
 c) Use your expression in 5 b) to calculate the cost for:
 (i) 40 guests. (ii) 50 guests. (iii) 55 guests.
 d) Use your expression in 5 b) to calculate how many guests would cost:
 (i) \$692. (ii) \$888. (iii) \$1042.

Number of Guests	Cost
25	
26	
•	
•	
•	
34	
35	

- 6 The graph you constructed in Activity 1 should look like this.

- a) Using the same grid, plot points in a different color to show the cost at Noble Pines for n guests for these values of n : 20, 25, 30, 40, 50, 60, 70. Join the new dots in your graph.
 b) Use your graph to check your answers to 5 c) and 5 d).
 c) Label your graphs “Galaxy Inn” and “Noble Pines.” Describe how the graphs are alike and how they are different.
 d) At what value of n do the graphs cross? What do you think this means?



- 7 Which location, the Galaxy Inn or the Noble Pines Country Club, do you think will offer the lowest total cost for the fundraising party? Explain.
- 8 Using your graphing calculator, display a table of values for Y_1 and Y_2 and discover how you can scroll through a table of values to find n for which $Y_1 = Y_2$. Then graph these functions on the screen and trace to the point of intersection to verify that $Y_1 = Y_2$ for $n = 30$.

X	Y ₁	Y ₂
30	510	510
31	527	524
32	544	538
33	561	552
34	578	566
35	595	580
36	612	594

X=30

UNIT 2

ANSWER KEY FOR ACTIVITY 2

- ① A flat fee of \$90 is a fee that is added after the number of guests is multiplied by the cost per guest. This fee is independent of the number of guests. A minimum charge of \$X, on the other hand, specifies that the total cost is the greater of \$X or the number of guests multiplied by the cost per guest. For a sufficiently large number of guests, the cost is merely the product of the number of guests and the cost per guest, and is not affected by the minimum charge.

- ② a) $\$14 \times 10 + \$90 = \$230$
 b) $\$14 \times 20 + \$90 = \$370$
 c) $\$14 \times 30 + \$90 = \$510$

- ③ a) The cost before the flat fee is
 $\$566 - \$90 = \$476$
 The number of guests is $476 \div 14 = 34$.
 34 guests would incur a total cost of \$566.
 b) The cost before the flat fee would be:
 $\$1000 - \$90 = \$910$
 The number of guests would be $910 \div 14 = 65$.
 65 guests would incur a total cost of \$1000.
 c) Answers will vary. Those students who have not learned how to solve linear equations algebraically are expected to use trial-and-error or to create a table of values of $14n + 90$ (see Exercise 4), beside a table of values of $17n$ and compare. It is important to provide these opportunities for students to explore, before teaching them the formal techniques of algebra. For students who have already learned to solve linear equations, we might expect something like this:

If n guests incur an average cost of \$17, then the total cost of n guests is $\$17n$. But the total cost of n guests at the Noble Pines Country Club is also given by $\$14n + 90$. These are two different ways to write the same total cost, so $17n = 14n + 90$. Solving this equation yields $n = 30$. That is, 30 guests at the Noble Pines Country Club would cost an average of \$17 per guest.

- ④ a) The table is shown here.

Number of Guests	Cost
25	440
26	454
27	468
28	482
29	496
30	510
31	524
32	538
33	552
34	566
35	580

- b) We observe that the cost of \$566 in the table is opposite 34 guests. This verifies the answer to ③ a).
 c) Comparing the two tables reveals that when there are 30 guests, the costs at the two locations are equal. But when the number of guests is 31 or more, Noble Pines costs less per guest than the Galaxy Inn.

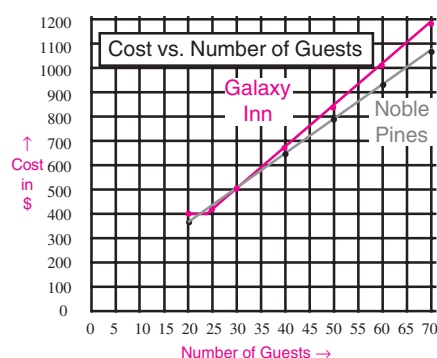
- ⑤ a) Answers will vary, but students should indicate that the number of guests should be multiplied by 14 and this product added to 90.

- b) Cost in dollars = $14n + 90$

- c) (i) $14 \times 40 + 90 = 650$
 (ii) $14 \times 50 + 90 = 790$
 (iii) $14 \times 55 + 90 = 860$

- d) (i) 43 guests (ii) 57 guests (iii) 68 guests

- ⑥ a) The graph should look something like this.



- c) Answers may vary. Students might observe that the graph for the Galaxy Inn is steeper (indicating higher per person cost). Also both graphs are straight lines (for $n \geq 25$).
 d) Students will observe that the graphs intersect at $n = 30$. From Exercise ④, they may know that this means the costs are equal for 30 guests.

- ⑦ Students should realize that Noble Pines offers the lower cost as long as there are more than 30 guests. Some may realize that Noble Pines also offers the lower cost when $n < 23$.

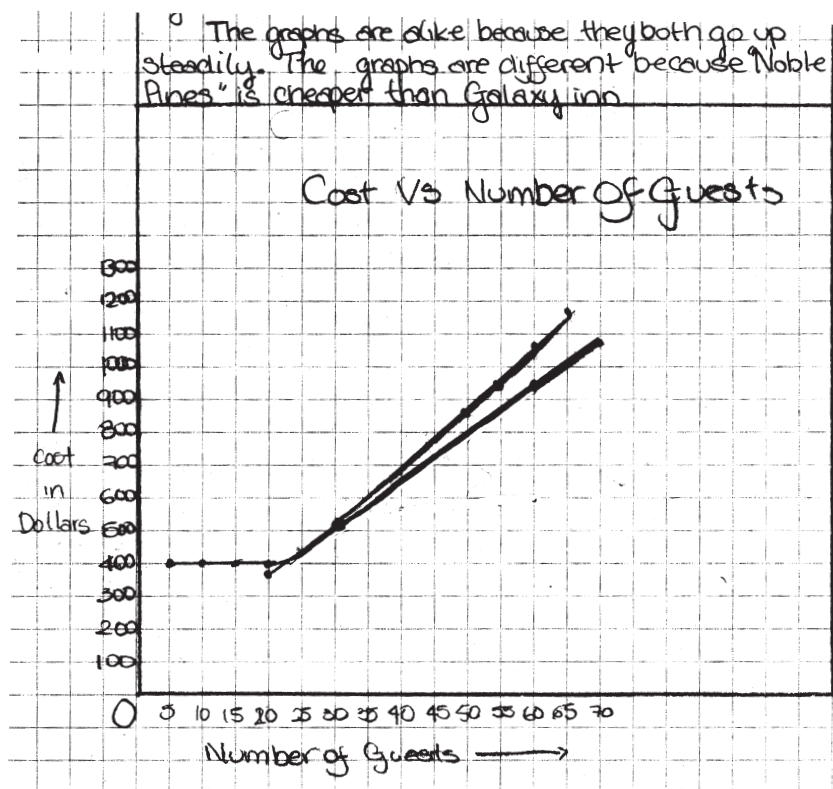
In the scoring guide below, the four levels span the spectrum of achievement from very low (Level 1) to exceptionally strong (Level 4). Level 3 represents “good performance” and each student who reaches this level is deemed competent to proceed to the next phase of instruction. Most students are expected to attain Level 3, and for this reason, the criteria in the Level 3 column are shaded and performance at other levels is sometimes expressed in terms of Level 3 achievement.

Scoring Guide for Activity 2				
	Level 1	Level 2	Level 3	Level 4
REPRESENTATION Representation of a Linear Function by: <ul style="list-style-type: none"> • a table • a linear equation • a graph (exercises ④ – ⑥) A3, A4, A7, A8	<ul style="list-style-type: none"> •Two or more of the three representations (table of values, equation, or graph) of the Noble Pines C. C. cost function are missing or incorrect. 	<ul style="list-style-type: none"> •One of the three representations (table of values, equation, or graph) of the Noble Pines C. C. is missing or incorrect. 	<ul style="list-style-type: none"> •The correct table of values, equation, and graph are given for the Noble Pines C. C. cost function, with a few minor errors. 	<ul style="list-style-type: none"> •The correct table of values, equation, and graph are given for the Noble Pines C. C. cost function, with almost no errors.
CONNECTIONS Interpretation of the Graph of a Linear Function or Equation (exercises ⑥ c), d) & ⑦) A4, A5, A6, A8	<ul style="list-style-type: none"> •There is no evidence that the student can interpret the different slopes of the graphs in terms of the costs they imply. 	<ul style="list-style-type: none"> •The student describes the difference in the slope of the two graphs but does not indicate an understanding that each of the locations offers a cheaper rate for different numbers of guests. 	<ul style="list-style-type: none"> •The steeper slope of the Galaxy Inn cost function is interpreted as a higher rate per person. •The student indicates that the Noble Pines C.C. is cheaper when $n > 30$. 	In addition to Level 3: <ul style="list-style-type: none"> •The student interprets correctly the intersection of two lines as the point of equal cost for the same number of guests. •The student recognizes that the Noble Pines is also cheaper when $n < 23$.

WHAT YOU MIGHT SEE

CONNECTIONS: INTERPRETATION OF THE GRAPH OF A LINEAR FUNCTION OR EQUATION

Level 3



c) Describe how the graphs are alike and how they are different.

d) At what value of n do the graphs cross? What do you think this means? I think it means

7. Which location, the Galaxy Inn or the Noble Pines Country Club, do you think will offer the lowest total cost for the fundraising party? Explain

I think that Noble Pines will offer the best price because if you look at the graph that we created you could tell that Noble Pines was cheaper than Galaxy Inn.

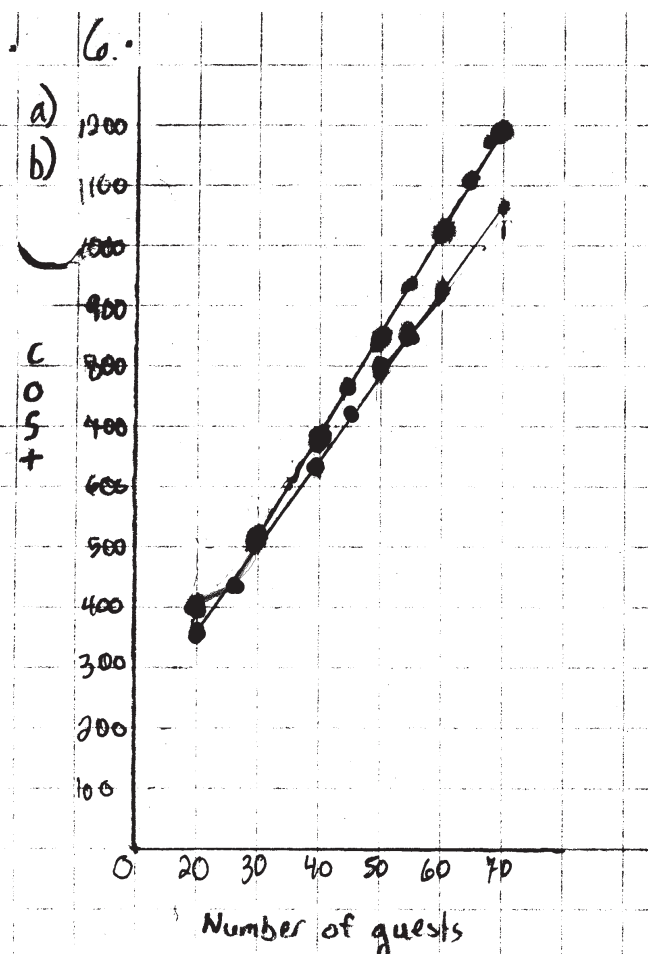
The Galaxy inn would probably be better for 23 people because - $23 \times 14 + 90 = 412$ - only by 12 dollars but it's still cheaper

The graphs of the cost functions for the Galaxy Inn and Noble Pines C. C. are done correctly. The student has observed that both graphs are increasing with n ("go up steadily") and that the Noble Pines C.C. offers the lower price. The student also observed that the Galaxy Inn was cheaper for $n = 23$, but stopped short of observing that it is cheaper for $23 \leq n < 30$. The student has discovered that the graphs intersect at $n = 30$, and approaches the correct interpretation of an intersection point as one that answers the question, "Is the cost ever the same for the same number of people?"

WHAT YOU MIGHT SEE

CONNECTIONS: INTERPRETATION OF THE GRAPH OF A LINEAR FUNCTION OR EQUATION

Level 4



Activity 2

6c) - after 30 guests both lines rise at fixed rate, but that rate is different for each hotel
 - both hotels are the same price for 30 guests
 - the Noble Pines always has a lower price, except for 30 guests because that price is the same

6d) the graphs cross at 30 guests, or \$510
 this means that there is a change in price at some point on the Galaxy Inn graph, because after 30 guests the rates go higher than Noble Pines

Ad.2.7. The Noble Pines Country Club offers the best deal for more than 30 guests, and less than 23. That's because of the Galaxy Inn's minimal charge of \$400. Anything less than \$400 at the Noble Pines is a better deal! BUT. The Galaxy Inn has a better deal for 24-30 guests. I'd still go with the Noble Pines, because there is a bigger range for a better deal

The graphs of the cost functions for the Galaxy Inn and Noble Pines C. C. are done correctly for 20 or more guests. (Students were not asked to extend the graphs to $n < 20$.) In the response to 6 c), the student has observed that both graphs are "rising at a fixed rate, but that rate is different for each hotel". The student also observes that Noble Pines C.C. is cheaper for $n > 30$. In 6 d), the student observes that the graphs cross at 30 guests and that this corresponds to a cost of \$510 at each location, but expresses it awkwardly as a "change in [average] price". The student has observed that the Galaxy Inn was cheaper for $23 \leq n < 30$, a fact that few students discovered. The student's response to 7 reveals a deep understanding of the idea that the estimated number of guests is subject to error and that Noble Pines is preferable because it offers the better option over a "bigger range" in the number of guests.

ACTIVITY 3 – TEACHER EDITION

COMPARING THE HOLIDAY LODGE WITH THE NOBLE PINES COUNTRY CLUB

Expectations Addressed

- A3** represent a variety of relations and functions with tables, graphs, verbal rules, and, when possible, symbolic rules;
- A4** develop a sound conceptual understanding of equation and of variable;
- A5** explore relationships between symbolic expressions and graphs, paying particular attention to the horizontal and vertical intercepts, points of intersection, and slope (for linear relations);
- A6** become fluent in generating equivalent expressions for simple algebraic expressions and in solving linear equations and inequalities;
- A7** use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships;
- A8** model and solve contextualized problems using various representations, such as graphs and tables, to understand the purpose and utility of each representation;
- A9** develop an initial understanding of rate of change, with emphasis on the connections among slope of a line, constant rate of change, and their meaning in context.

Context

This activity extends the investigation of linear functions in Activity 2 from the exploration of the cost functions $17n$ and $14n + 90$ to the cost function $12(n - 25) + 500$. The lesson development parallels that of Activity 2. After students explain how to compute the costs for various numbers of guests at the Holiday Lodge, they are asked in Exercise 5b) to translate this verbal description into an algebraic expression. In Exercise 6, students graph the linear function $12(n - 25) + 500$ on the grid used in Activities 1 and 2. This enables them to compare the graphs of the cost functions of the three sites. In Exercise 6 d), students are asked to indicate the value of n at which the graphs of the Noble Pines C.C. and the Holiday Lodge cross. They are expected to observe that the graphs cross at $n = 55$, indicating that the costs for both sites are equal for this number of guests. This is equivalent to solving graphically the linear equation $14n + 90 = 12(n - 25) + 500$. A formal algebraic solution of this equation is not expected.

The purpose of Activity 3 is to help students create and interpret the tabular and the graphical representations of linear functions in both simplified and unsimplified forms. They will observe that a minimum cost makes that option a more expensive proposition for small values of n , but for a sufficiently large value of n , the per person cost (coefficient of n) is more important. Another objective of this lesson is to help students use inequalities to determine the range in the values of n for which each option is preferred. The cartoon on page 78 is intended to emphasize this point. That is, as the number of guests increases, it is the per person cost (coefficient of n) that has the dominant effect on the total cost, and flat fees and minimum charges become less important.

ACTIVITY 3 – STUDENT PAGE

COMPARING THE HOLIDAY LODGE WITH THE NOBLE PINES COUNTRY CLUB



DISCUSS

1. a) Explain what is meant by the statement, "We charge \$500 for the first 25 guests and then \$12 per guest for those in excess of 25"?
- b) What would the Holiday Lodge charge if only 23 guests attended the party? What would the charge be for 26 guests?

ACTIVITY 3 – TEACHER EDITION

The Lesson Launch 15 minutes

Before launching this lesson, ask whether anyone was successful in solving the problem handed out at the end of the previous class (problem 2, p. 18). Display a copy of the problem on the overhead projector and discuss what it means when a two-pan scale is in balance. Ask students,

- Give an algebraic expression for the total mass of the coins on the left pan; on the right pan. [Some students will need help distinguishing between the meanings of $12(n - 25)$ and $12n - 25$.]
- The two pans are in balance; what does that tell us?
- How would you state this algebraically?
- Finding the value of n that makes both sides equal is called “solving for n ”; how could we do this?

Allow the students to struggle with the last question before providing a hint such as, *If we subtract equal quantities from both sides of the equation, are the remaining amounts still equal? How do you know?* Explain to students that they will be solving the problem in a different way as they work through this activity.

To launch the lesson, distribute student pages 78 and 79 and have students read to themselves the cartoon on page 78. Then ask students why the boy’s suggestion is not helpful. While his observation is true, it does not tell us how large n must be to make the Holiday Lodge the most economical choice.

Paired Activity 30 minutes

Group the students in the same pairs as for Activity 2. Have each pair complete Exercises 1 through 5 with each student recording the answers in his notebook. Ask students, when finished, to complete Exercise 6 on the grid they used in Activity 2. When almost all students are finished, ask about the value of n at which the graphs for Holiday Lodge and Noble Pines cross. Ask what this means about the costs of the two different sites. Ensure that students understand not only that this is the value of n for which the two sites have the same cost, but also that as the number of guests is increased beyond this, the Holiday Lodge offers the lower cost. If students have access to a graphing calculator, ask them to define $Y_2 = 14X + 90$ and $Y_3 = 12(X - 25) + 500$ and then display a table of values for Y_2 and Y_3 . Have them scroll through the table to check their answers to Exercises 5 c) and 6 d). Ask them to graph these functions and trace along the graphs to check their answers to Exercise 6 d).

Closure

If you have a graphing calculator viewscreen, display a table of values for Y_2 and Y_3 and show how we can scroll through a table of values to find n for which $Y_2 = Y_3$. Then graph these functions on the screen and trace to the point of intersection to verify that $Y_2 = Y_3$ for $n = 55$. Discuss why solving for n to make $Y_2 = Y_3$ is the same problem as the two-pan balance problem 2 presented above. Distribute copies of p. 93 and discuss the solutions.

ACTIVITY 3 – STUDENT PAGE

COMPARING THE HOLIDAY LODGE WITH THE NOBLE PINES COUNTRY CLUB

All the exercises in this activity pertain to the quote from the Holiday Lodge.

- How much would the Holiday Lodge charge for:
 - 20 guests?
 - 30 guests?
 - 40 guests?
 - How many guests would incur a total cost of \$824?
 - How many guests would it take to give an average cost per guest of \$17?
 - Is there a number of guests that would result in an average cost per guest of \$18? Give reasons for your answer.
 - Make a table showing the cost for each number of guests between 50 and 60.
 - Use your table to check your answer to 2a).
 - Modify the table you constructed in Exercise 4 of Activity 2 to show the cost of the Noble Pines Country Club for between 50 and 60 guests. How many guests are needed to make Holiday Lodge cost per guest less than the Noble Pines Country Club?
 - Describe how you could compute the cost for any given number of guests.
 - Let n represent the number of guests attending the party. Write an expression for the cost for n guests.
 - Use your expression in 5 b) to calculate the cost for:
 - 40 guests.
 - 50 guests.
 - 55 guests.
 - Use your expression in 5 b) to calculate how many guests would cost:
 - \$716.
 - \$836.
 - \$944.
 - On the graph you created in Exercise 6 of Activity 2, plot points in a different color showing the cost of n guests for these values of n : 30, 35, 40, 45, 50, 55, 60.
 - Join the dots to make a graph. Label your graph “Holiday Lodge”.
 - Describe how all the graphs are alike and how they are different.
 - At what value of n do the Noble Pines C. C. and Holiday Lodge graphs cross? What do you think this means?
- **SAVE YOUR GRAPH FOR ACTIVITY 4.**
- Which location, the Noble Pines Country Club or the Holiday Lodge, do you think will offer the lowest total cost for the fundraising party? Explain.

Number of Guests	Cost
50	
51	
•	
•	
•	
59	
60	

X	Y ₂	Y ₃
33	860	860
36	874	872
39	888	884
42	902	896
45	916	908
48	930	920
51	944	932
X=55		

ACTIVITY 3 – STUDENT PAGE

COMPARING THE *HOLIDAY LODGE* WITH THE *NOBLE PINES COUNTRY CLUB*



DISCUSS

- 1 a) Explain what is meant by the statement, "We charge \$500 for the first 25 guests and then \$12 per guest for those in excess of 25"?
- b) What would the Holiday Lodge charge if only 23 guests attended the party? What would the charge be for 26 guests?

COMPARING THE HOLIDAY LODGE WITH THE NOBLE PINES COUNTRY CLUB

All the exercises below pertain to the quote from the Holiday Lodge.

- ② How much would the Holiday Lodge charge for:
 a) 20 guests? b) 30 guests? c) 40 guests?
- ③ a) How many guests would incur a total cost of \$824?
 b) How many guests would it take to give an average cost per guest of \$17?
 c) Is there a number of guests that would result in an average cost per guest of \$18? Give reasons for your answer.

- ④ a) Make a table showing the cost for each number of guests between 50 and 60.
 b) Use your table to check your answer to Exercise ③ a).
 c) Modify the table you constructed in Exercise ④ of Activity 2 to show the cost of the Noble Pines Country Club for between 50 and 60 guests. How many guests are needed to make Holiday Lodge cost per guest less than the Noble Pines Country Club?

Number of Guests	Cost
50	
51	
•	
•	
•	
59	
60	

- ⑤ a) Describe how you could compute the cost for any given number of guests.
 b) Let n represent the number of guests attending the party. Write an expression for the cost for n guests.
 c) Use your expression in ⑤ b) to calculate the cost for:
 (i) 40 guests. (ii) 50 guests. (iii) 55 guests.
 d) Use your expression in ⑤ b) to calculate how many guests would cost:
 (i) \$716. (ii) \$836. (iii) \$944.

- ⑥ a) On the graph you created in Exercise ⑥ of Activity 2, plot points in a different color showing the cost of n guests for these values of n : 30, 35, 40, 45, 50, 55, 60.
 b) Join the dots to form a graph. Label your graph “Holiday Lodge.”
 c) Describe how all the graphs are alike and how they are different.
 d) At what value of n do the Noble Pines C. C. and Holiday Lodge graphs cross? What do you think this means? ➤ **SAVE YOUR GRAPH FOR ACTIVITY 4.**

- ⑦ Which location, the Noble Pines Country Club or the Holiday Lodge, do you think will offer the lowest total cost for the fundraising party? Explain.

- ⑧ Using your graphing calculator, display a table of values for Y_2 and Y_3 and discover how to scroll through a table of values to find n for which $Y_2 = Y_3$. Then graph these functions on the screen and trace to the point of intersection to verify that $Y_2 = Y_3$ for $n = 55$.

X	Y_2	Y_3
55	860	860
56	874	872
57	888	884
58	902	896
59	916	908
60	930	920
61	944	932

X=55

UNIT 2

ANSWER KEY FOR ACTIVITY 3

- ① a) There is a minimum charge of \$500, and after that charge, the cost for each guest from the 26th on is \$12. In mathematical terms, if $n \leq 25$, the cost is \$500. When $n > 25$, the cost is \$12 for each additional guest beyond the first 25 plus the initial \$500.

b) 23 guests would cost \$500, and 26 guests would cost $\$500 + \$12 = \$512$.

- ② a) \$500
b) $\$500 + \$12 \times 5 = \$560$
c) $\$500 + \$12 \times 15 = \$680$

- ③ a) The cost for the guests beyond the first 25 is $\$824 - \$500 = \$324$. This represents $324 \div 12$ or 27 guests beyond the first 25. So \$824 would be the cost of 25 + 27 or 52 guests.

b) Answers will vary. Students who have not learned how to solve linear equations algebraically are expected to use trial-and-error or to create a table of values of $500 + 12(n - 25)$ (see Exercise ④), beside a table of values of $17n$ and compare. It is important to provide these opportunities for students to explore before teaching them the formal techniques of algebra. An exceptional student might graph both these functions and look for a point of intersection. For those who have already learned to solve linear equations, we might expect something like this:

If n guests incur an average cost of \$17, then the total cost of n guests is $\$17n$. But the total cost of n guests at the Holiday Lodge is also given by $\$500 + \$12(n - 25)$. These are two different ways to write the same total cost, so:

$$17n = 500 + 12(n - 25).$$

Solving this equation yields $n = 40$. That is, 40 guests at the Noble Pines Country Club would cost an average of \$17 per guest.

c) Any of the methods described in Part b) could be used to determine that there is no integral value for n for which the expressions $500 + 12(n - 25)$ and $18n$ assume the same value. Depending on the method used, students may indicate that the average cost per guest at the Holiday Lodge is greater than \$18 for $n \leq 33$ and less than \$18 for $n > 33$.

- ④ a) This is the table.

b) We observe that the cost of \$824 in the table is opposite 52 guests. This verifies the answer to ③ a).

c) Comparing the two columns reveals that the costs are equal for 55 guests. But when the number of guests is 56 or more, the Holiday Lodge cost per guest is less than that of Noble Pines.

Number of Guests	Cost at Holiday Lodge	Cost at Noble Pines
50	800	790
51	812	804
52	824	818
53	836	832
54	848	846
55	860	860
56	872	874
57	884	888
58	896	902
59	908	916
60	920	930

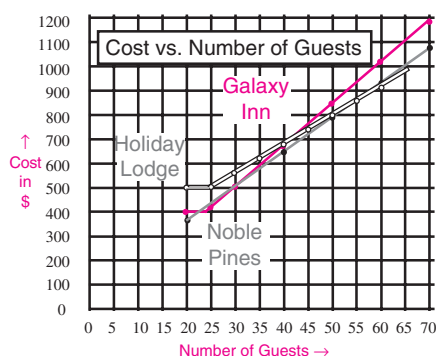
- ⑤ a) Answers will vary, but students should indicate that 25 is to be subtracted from the number of guests, and the difference should be multiplied by 12. Then 500 should be added to the result.

b) Cost in dollars = $12(n - 25) + 500$ for $n > 25$ and \$500 for $n \leq 25$.

- c) (i) $\$12 \times (40 - 25) + \$500 = \$680$
(ii) $\$12 \times (50 - 25) + \$500 = \$800$
(iii) $\$12 \times (55 - 25) + \$500 = \$860$

d) (i) 43 guests (ii) 53 guests (iii) 62 guests

- ⑥ a) & b) The graph should look something like this.



c) Answers may vary. Students might observe a difference in steepness (slope). Also all graphs are straight lines for sufficiently large n .

d) Students will observe that the graphs intersect at $n = 55$. From Exercise ④, they may know that this means the costs are equal for 55 guests.

- ⑦ Students should recognize from the graph or the table that the Holiday Lodge offers the lower cost for more than 55 guests.

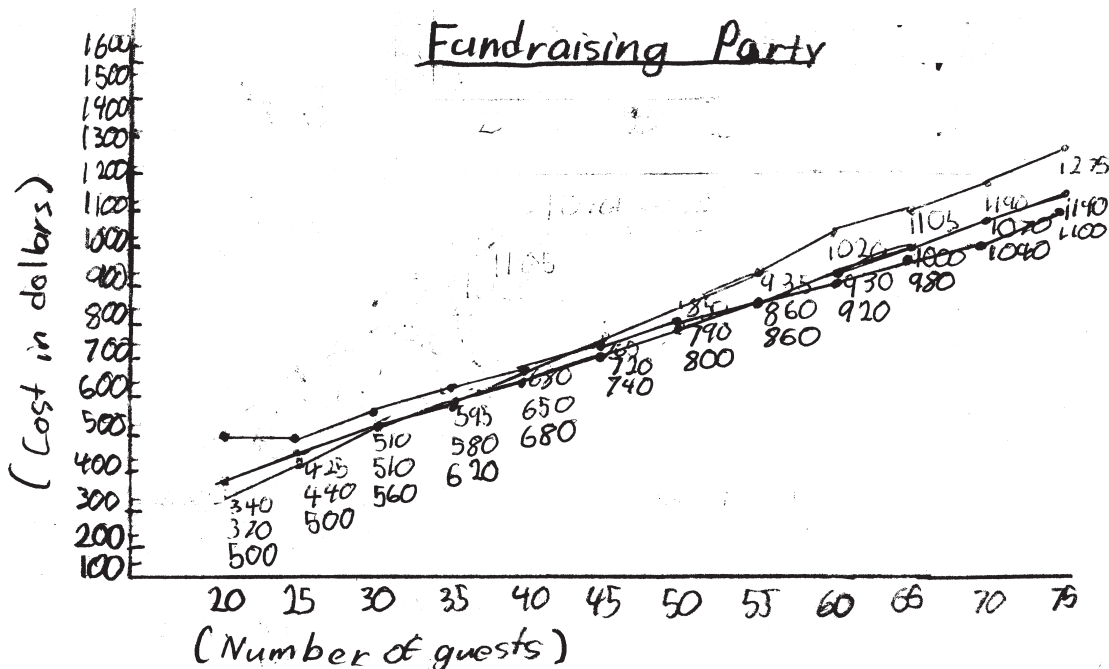
In the scoring guide below, the four levels span the spectrum of achievement from very low (Level 1) to exceptionally strong (Level 4). Level 3 represents “good performance” and each student who reaches this level is deemed competent to proceed to the next phase of instruction. Most students are expected to attain Level 3, and for this reason, the criteria in the Level 3 column are shaded and performance at other levels is sometimes expressed in terms of Level 3 achievement.

Scoring Guide for Activity 3				
	Level 1	Level 2	Level 3	Level 4
PROBLEM SOLVING				
Solution of Problems by Applying an Appropriate Strategy (exercises 3 , 4 , 5 , 7) A6, A7, A8	<ul style="list-style-type: none"> •Applies a limited range of strategies to the solution of problems and frequently makes errors. 	<ul style="list-style-type: none"> •Usually solves problems using an appropriate strategy but with several minor errors. 	<ul style="list-style-type: none"> •Solves problems using an appropriate strategy almost always and with few minor errors. 	<ul style="list-style-type: none"> •Always solves problems using an appropriate strategy and sometimes employs a creative new strategy.
REPRESENTATION				
Representation of a Linear Function by: <ul style="list-style-type: none"> • a table • a linear equation • a graph (exercises 4 – 6) A3, A4, A7, A8	<ul style="list-style-type: none"> •Two or more of the three representations (table of values, equation, or graph) of the Holiday Lodge cost function are missing or incorrect. 	<ul style="list-style-type: none"> •One of the three representations (table of values, equation, or graph) of the Holiday Lodge cost function is missing or incorrect. 	<ul style="list-style-type: none"> •The correct table of values, equation, and graph are given for the Holiday Lodge cost function, with a few minor errors. 	<ul style="list-style-type: none"> •The correct table of values, equation, and graph are given for the Holiday Lodge cost function, with almost no errors.
CONNECTIONS				
Solution of a Linear Equation <ul style="list-style-type: none"> • Algebraically • Graphically (exercises 5 d) & 6 b) A5, A6, A8	<ul style="list-style-type: none"> •There is no evidence of a correctly constructed linear equation corresponding to the cost of a particular number of guests. 	<ul style="list-style-type: none"> •The appropriate equation is constructed, but there is evidence of difficulty in solving it to obtain the number of guests corresponding to a given cost. 	<ul style="list-style-type: none"> •Determines algebraically the number of guests corresponding to given costs at the Holiday Lodge and verifies graphically the algebraic solution of a linear equation, with few minor errors. 	<ul style="list-style-type: none"> •And in addition to Level 3, interprets correctly the intersection of two lines as the point of equal cost for the same number of guests.

WHAT YOU MIGHT SEE

PROBLEM SOLVING: SOLUTION OF PROBLEMS BY APPLYING AN APPROPRIATE STRATEGY

Level 1



This student drew the graphs correctly and shows some competence in solving one-step problems. For example, the student answered Exercises ①, ②, ③ a), and ⑤ c) correctly, using the tables of values. However, the student seems unable to apply an appropriate strategy to the solution of more difficult problems, such as ③ b) and c). To determine how many people it would take to average \$17 per person at Holiday Lodge, the student divided \$500 by 17 to obtain the estimate 27.7. Also, in Exercise ④ c), the student was unable to determine, even from the tables of values, the number of guests that would make the Holiday Lodge the less expensive option. In Exercise ⑦, the student correctly chooses the Holiday Lodge as the cheaper option, but does so by comparing costs for 60 guests—a single one-point comparison.

- ③ a) How many guests would incur a total cost of \$824? 52 guests
 b) How many guests would it take to reduce the average cost per guest to \$17? 29 guests
 c) Is there a number of guests that would result in an average cost per guest of \$18? Give reasons for your answer. 27 guests, because $500 \div 17 = 27.7$

- ④ a) Make a table showing the cost for each number of guests between 50 and 60. ✓
 b) Use your table to check your answer to Exercise ③. a). ✓
 c) Modify the table you constructed in Exercise ④ of Activity 2 to show the cost of the Noble Pines Country Club for between 50 and 60 guests. How many guests are needed to make Holiday Lodge cost per guest less than the Noble Pines Country Club? 46 guests

Number of Guest	Cost
50	850
51	860
52	870
53	880
54	890
55	900
56	910
57	920
58	930
59	940
60	950

- ⑤ a) Describe how you could compute the cost for any given number of guests.
 b) Let n represent the number of guests attending the party. Write an expression for the cost for n guests. $n \times 25 \times 12 + 500 =$
 c) Use your expression in Part b) to calculate the cost for:
 (i) 40 guests \$650 (ii) 50 guests \$800 (iii) 55 guests \$920
- ⑥ a) On the graph you created in Exercise ⑥ of Activity 2, plot points in a different colour showing the cost of n guests for these values of n : 30, 35, 40, 45, 50, 55, 60. ✓
 b) Join the dots to form a graph. Label your graph "Holiday Lodge". ✓
 c) Describe how all the graphs are alike and how they are different.
 d) At what value of n do the Noble Pines C. C. and Holiday Lodge graphs cross? What do you think this means? \$510 I am not quite sure, but I know it means that everyone in excess of 30 guests are getting a better deal at Noble Pines
- ⑦ Which location, the Noble Pines Country Club or the Holiday Lodge, do you think will offer the lowest total cost for the fundraising party? Explain. I think the holiday lodge would be better, because,

$$HL = 60gs = 920$$

$$NP = 60gs = 1020$$

WHAT YOU MIGHT SEE

PROBLEM SOLVING: SOLUTION OF PROBLEMS BY APPLYING AN APPROPRIATE STRATEGY

Level 4

Not only did this student answer all of the exercises correctly, but she shows some flamboyance in her answer to Exercise 3 c). She determines that 33 guests yield an average cost of \$17.88 and 34 guests yield an average cost of \$18.06, and then asserts ("Since you can't divide a guest into pieces...") it is not possible to attain an average cost per guest of exactly \$18.00.

In Exercise 5, the student not only describes correctly the algorithm for calculating the cost function, but she gives a correct algebraic expression to define it, and applies it correctly in her calculations.

Finally, in Exercise 7, she is able to describe correctly the range of values of n for which each location is the less expensive option. In general, this student's responses indicate that she understands both the algebraic and graphical representations of the cost functions and is able to apply them in the solution of problems.

3.c. No, not exactly, because 33 guests makes the average cost \$17.88 and 34 guests makes the average cost \$18.06. Since you can't divide a guest into pieces, you will never get exactly \$18.00 for an average cost.

4.a.

Number of Guests	Cost (in dollars)
50	\$800
51	\$812
52	\$824
53	\$836
54	\$848
55	\$860
56	\$872
57	\$884
58	\$896
59	\$908
60	\$920

HOLIDAY LODGE

Number of Guests	Cost (in dollars)
50	\$790
51	\$804
52	\$818
53	\$832
54	\$846
55	\$860
56	\$874
57	\$888
58	\$902
59	\$916
60	\$930

NOBLE PINES

5.a. If the number of guests is less or exactly 25, the charge is \$500.00. If the number of guests is 26 or more, you subtract 25 and then multiply by 12. Finally, you add 500. The answer is the cost, in dollars.

6. a) Describe how you could compute the cost for any given number of guests. (on lined paper)

b) Let n represent the number of guests attending the party. Write an expression for the cost for n guests (when $n > 25$) $\rightarrow (n - 25)12 + 500$ P9.4

c) Use your expression in Part b) to calculate the cost for:

(i) 40 guests.

(ii) 50 guests.

(iii) 55 guests.

\$680.00

\$800.00

\$860.00

6. a) On the graph you created in Exercise 6 of Activity 2, plot points in a different colour showing the cost of n guests for these values of n : 30, 35, 40, 45, 50, 55, 60.

b) Join the dots to form a graph. Label your graph "Holiday Lodge". (on blank paper.) P9.5

c) Describe how all the graphs are alike and how they are different.

d) At what value of n do the Noble Pines C. C. and Holiday Lodge graphs cross?

What do you think this means? When $n = 55$. I think this means that after this point, Holiday Lodge runs lower than Noble Pines, which means that when $n \geq 55$, Holiday Lodge is cheaper.

7. Which location, the Noble Pines Country Club or the Holiday Lodge,

do you think will offer the lowest total cost for the fundraising party? Explain.

I think Holiday Lodge, because the price is the cheapest for 70 people. Noble Pines is the best location, if there are less than 55 guests, but if there are more, (as in Jennifer's case), Holiday Lodge is the best choice, in terms of looking at price.

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